PROVING ANGLE CONJECTURES

Vertical Angle Conjecture. Two vertical angles are congruent.

![Diagram of vertical angles](image)

Figure 16.29

**Proof.** Sum of each pairs of angles ($\angle DMA$, $\angle AMC$) and ($\angle AMC$, $\angle BMC$) is equal to $180^\circ$ (Figure 16.29). That is,

1. $m\angle AMC + m\angle AMD = 180^\circ$.
2. $m\angle AMC + m\angle BMC = 180^\circ$.

Compare (1) and (2):

3. $m\angle AMC + m\angle AMD = m\angle AMC + m\angle BMC$.

Canceling $m\angle AMC$ out of the both sides of (3), we get:

4. $m\angle AMD = m\angle BMC$.

**Triangle Sum Conjecture.** The sum of the interior angles in a triangle is $180^\circ$. 


**Proof.** In Figure 16.30, \( \triangle ABC \) is given. We want to prove that

\[
(1) \quad m\angle BAC + m\angle B + m\angle C = 180^\circ.
\]

Draw segment \( \overline{MD} \) through point \( A \) parallel to \( \overline{BC} \). By property of alternate interior angles, we have:

\[
(2) \quad m\angle MAB = m\angle B
\]

\[
(3) \quad m\angle DAC = m\angle C.
\]

We have

\[
(4) \quad m\angle MAB + m\angle BAC + m\angle DAC = 180^\circ.
\]

Replace (2) and (3) in (4):

\[
(5) \quad m\angle B + m\angle BAC + m\angle C = 180^\circ.
\]

**Exterior Angle Conjecture.** The measure of an exterior angle in a triangle is equal to the sum of the two remote interior angles.

**Proof.** In Figure 16.31, \( \triangle ABD \) is given. We want to prove that

\[
(1) \quad m\angle ADC = m\angle A + m\angle B.
\]

We know that

\[
(2) \quad m\angle ADC + m\angle ADB = 180^\circ.
\]
(3) \[ m\angle A + m\angle B + m\angle ADB = 180^\circ. \]

Comparing (2) and (3),

(4) \[ m\angle ADC + m\angle ADB = m\angle A + m\angle B + m\angle ADB. \]

Canceling \( m\angle ADB \) out from the both sides of (4),

(5) \[ m\angle ADC = m\angle A + m\angle B. \]

![Figure 16.31](image_url)

**Practice.** Find \( m\angle A \) in Figure 16.32.
Solution. \( \angle DBE \) and \( \angle ABC \) are vertical angles. Therefore,

\[
(1) \quad m \angle DBE = m \angle ABC = 35^\circ.
\]

Also,

\[
(2) \quad m \angle FCB = m \angle A + m \angle CBA.
\]

Replace the known values in (2):

\[
(3) \quad 85^\circ = m \angle A + 35^\circ.
\]

Solve equation (3):

\[
(4) \quad m \angle A = 50^\circ.
\]

Practical Exercise 3. Find \( m \angle B \) in Figure 16.33.

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**Figure 16.32**

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**Figure 16.33**