POSTULATES OF GEOMETRY

Postulates play very important roles in solving problems or proofing theorems. Rarely can you find a problem or theorem in which postulates are not used. Here are the geometric postulates along with their graphs.

Points, Lines, Planes, and Angles Postulates

Line Postulate 1. Every two points are on a line (Figure 16.1).

![Figure 16.1]

Line Postulate 2. Every line has at least two points (Figure 16.1)

Betweenness Postulate. Three points A, B, and C are given on a line. Then one of the points is between the other two points (Figure 16.2).

![Figure 16.2]

Plane Postulate. Every three noncollinear points are on a plane. These points are called coplanar (Figure 16.3).

![Figure 16.3]
Plane-Line Postulate. A plane contains two points of a line. Then the line is on the plane (Figure 16.4).

Two Planes Postulate. The common part of two intersecting planes is a line (Figure 16.5).

Space Postulate. There are at least four noncoplanar points in space.

Ruler Postulate. Any point on a real axis corresponds to a real number. The distance between two points on a real axis equals to the absolute value of the difference of the corresponding numbers (Figure 16.6).
**Protractor Postulate.** A ray $\overline{MN}$ and a number $t$ between 0 and 180 are given. Then there is exactly one ray $\overline{MA}$ such that $m\angle AMN = t^\circ$ (Figure 16.7).

**Segment Addition Postulate.** Let $M$ be between $A$ and $B$ on a line. Then $AM + MB = AB$ (Figure 16.8). Let $P$, $M$, and $N$ be on a line. If $PM + MN = MN$, then $M$ is between $P$ and $N$.

**Parallel Lines Postulate 1.** A point $A$ and a line $k$ are given. Point $A$ is not on $k$. Then there exists exactly one line through $A$ parallel to $k$ (Figure 16.9).
Corresponding Angles Postulate. Two parallel lines cut by a line. Then the corresponding angles are equal. In Figure 16.10, lines \( j \) and \( k \) are parallel. Then, \( m \angle b = m \angle n \), \( m \angle d = m \angle p \), \( m \angle a = m \angle m \), and \( m \angle c = m \angle q \).

![Figure 16.10](image)

Parallel Lines Postulate 2. Two lines cut by a transversal. If a pair of corresponding angles is congruent, then the lines are parallel.

Alternate Interior Angles Postulate. Two parallel lines cut by a transversal. Then each pair of the alternate interior angles is equal. In Figure 16.11, lines \( s \) and \( t \) are parallel. Then, \( m \angle a = m \angle d \) and \( m \angle b = m \angle c \).

![Figure 16.11](image)

Triangles Postulates

SSS Postulate. All the sides of a triangle are equal to their corresponding sides in another triangle. Then the two triangles are congruent. In Figure 16.12, \( AB = DE \), \( AC = DF \), and \( BC = EF \). Then, \( \triangle ABC \) and \( \triangle DEF \) are congruent.
SAS Postulate. Two sides and the included angle from one triangle are equal to their corresponding sides and angle in another triangle. Then the two triangles are congruent. In Figure 16.13, $AB = DE$, $m\angle A = m\angle D$, and $AC = DF$. Then $\triangle ABC$ and $\triangle DEF$ are congruent.

ASA Postulate. Two angles and the common side of these angles in a triangle are equal to their corresponding side and angles in another triangle. Then the
two triangles are congruent. In Figure 16.14, \( m \angle A = m \angle D, \ AB = DE, \) and \( m \angle B = m \angle E. \) Then, \( \triangle ABC \) and \( \triangle DEF \) are congruent.

**Figure 16.14**

**AAS Postulate.** Two angles and a nonincluded side of a triangle are equal to their corresponding angles and side in another triangle. Then the triangles are congruent. In Figure 16.15, \( m \angle A = m \angle D, \ m \angle C = m \angle E, \) and \( BC = EF. \) Then, \( \triangle ABC \) and \( \triangle DEF \) are congruent.
**HL Postulate.** A leg and the hypotenuse of a right triangle are equal to their corresponding sides in another right triangle. Then the two triangles are congruent. In Figure 16.16, $\angle A = \angle E = 90^\circ$, $DE = AB$, and $DF = BC$. Then, $\triangle ABC$ and $\triangle DEF$ are congruent.
**AH Postulate.** The hypotenuse and an acute angle of a right triangle are equal to their corresponding elements in another right triangle. Then the two triangles are congruent (Figure 16.17).

![Diagram of AH Postulate]

**AA Similarity Postulate.** Two angles of a triangle are equal to the two angles of another triangle. Then, the two triangles are similar. In Figure 16.18, \( m \angle A = m \angle D \) and \( m \angle C = m \angle F \). Then \( \triangle ABC \) and \( \triangle DEF \) are similar.
SSS Similarity Postulate. The sides of a triangle are proportional to the sides of the other triangle. Then the two triangles are similar. In Figure 16.19, we have

\[
\frac{EF}{PR} = \frac{DE}{PQ} = \frac{DF}{QR}.
\]

Then, \(\triangle DEF\) and \(\triangle PQR\) are similar.
SAS Similarity Postulate. Two sides of a triangle are proportional to the two sides of other triangle. The angles included by each pair of these sides are equal. Then the triangles are similar. In Figure 16.20,

\[ \frac{EF}{PR} = \frac{DE}{PQ} \text{ and } m\angle E = m\angle P. \]

Then, \( \triangle DEF \) and \( \triangle PQR \) are similar.

Addition Postulates

Angle Addition Postulate. Let \( M \) be a point inside \( \angle ABC \) (Figure 16.21). Then, 
\[ m\angle ABM + m\angle MBC = m\angle ABC. \]
For a point $D$ and a given angle $\angle PQR$, $m\angle PQD + m\angle DQR = m\angle PQR$. Then, $D$ is an interior point of $\angle PQR$.

**Arc Addition Postulate.** Point $A$ is between points $M$ and $N$ on a circle. Then, $\overset{\frown}{AM} + \overset{\frown}{MB} = \overset{\frown}{AB}$ (Figure 16.22).